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Conditional Beta-Normal Modeling of Risk Interactions in Financial Stability Dynamics

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ABSTRACT

This study models the relationship between loan-to-deposit ratio (LDR) and liquidity ratio (LR) using a conditional Beta-Normal distribution (CBND). LDR follows a Normal Distribution, while LR follows a Beta distribution with shape parameters dependent on LDR. Parameters were estimated using Maximum Likelihood Estimation (MLE). Results reveal that higher LDR values lead to more concentrated LR distributions, reflecting the nonlinear dependence between these financial indicators. A Mean Squared Error (MSE) value of 0.6922 demonstrates the model's adequacy. The CBND framework captures skewness, nonlinearity; and asymmetry in the joint behaviour of financial stability indicators, providing valuable insights for liquidity management and regulatory assessment in banking systems.

KEYWORDS

Beta-Normal, Conditional Distribution, Loan-to-Deposit Ratio, Liquidity Ratio, Dependence Structure, Financial-Stability.

1. Introduction

Univariate data are frequently encountered across diverse research domains. The pursuit of distributions capable of effectively describing such data has led to both classical and contemporary models such as the Normal, Gamma, Weibull, and Beta families which continue to be refined and applied today ([2], [5]). These distributions are widely used to model physical, biological, economic, and financial variables in both theoretical and applied contexts. In the bivariate context, the bivariate normal distribution remains a common choice due to its simplicity and tractability. However, it often fails to represent the distribution of bounded or asymmetric data. To overcome this, several alternative models have been proposed, including copula-based methods ([9], [3]), conditional specification techniques ([13], [8]), and probabilistic integral transformations ([6]). These models are particularly useful for combining random variables defined on different support ranges. Other researchers have applied linear regression ([11]), panel models ([7]), and Beta regression ([12], [4]) to study the loan-to-deposit ratio (LDR) and liquidity ratio (LR). [1] employed a Factor-Augmented Vector Autoregression (FAVAR) model using Nigerian banking data to evaluate how changes in the LDR policy affect system-wide liquidity. Their approach captured both direct and latent macro-financial shocks influencing liquidity behavior. [10] utilized panel regression and correlation analysis to assess the impact of LDR and liquidity ratio on the financial performance of Nigerian deposit money banks, controlling for capital adequacy and asset

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quality. Meanwhile, [14] adopted an explanatory research design with panel least squares regression to examine the effect of liquidity-ratio requirements and LDR thresholds on the profitability and liquidity performance of Kenyan commercial banks. Collectively, these studies apply robust multivariate and panel estimation frameworks that confirm a statistically significant yet nonlinear relationship between LDR and liquidity ratio, thereby justifying the need for a conditional probabilistic modeling approach such as the Conditional Beta-Normal Distribution (CBND) used in the present study. It focuses on modeling financial indicators where the liquidity ratio (LR) is a bounded variable (between 0 and 1) i.e. proportion-based, while the loan-to-deposit ratio (LDR) is continuous and unbounded (ratio-based). In practice, LDR values greater than 1 indicate over-lending or liquidity stress, a common financial reality. The phrase "unbounded" does not imply infinite range but reflects that LDR is not mathematically restricted to 0–1, and can exceed 100% when loans surpass deposits.

2. Material and Method

2.1. *Conditional Beta-Normal Distribution (CBND)*

In financial studies, it is well established that the Normal distribution aptly describes unbounded and symmetric financial indicators such as the loan-to-deposit ratio (LDR) ([9]). On the other hand, bounded ratios such as the liquidity ratio (LR) are often effectively modeled using the Beta distribution due to its flexibility over the unit interval ([13], [12]). Consequently, it is reasonable to explore the use of a bivariate conditional Beta–Normal distribution to jointly model LDR and LR. This approach can capture the asymmetry and bounded nature of the LR while conditioning on the LDR.

Let X denote the Loan-to-Deposit Ratio (LDR), modeled as normally distributed with location parameter μ and scale parameter σ:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \quad -\infty < x < \infty. \tag{1}$$

Let Y denote the Liquidity Ratio (LR), which given $X = x$, follows a Beta distribution with shape parameters α and β:

$$f_{Y|X}(y|x) = \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)}, \quad 0 < y < 1, \tag{2}$$

where $B(\cdot)$ denotes the Beta function.

2.1.1. Joint Probability Density Function

A joint probability density function called conditional beta-normal distribution (CBND) for X and Y can be derived by multiplying equation (1) and (2) using the relation

$$f_{X,Y}(x,y) = f_X(x) \cdot f_{Y|X}(y|x). \tag{3}$$

CBND is therefore given by

$$f_{X,Y}(x,y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \cdot \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)}, \quad -\infty < x < \infty,\ 0 < y < 1. \tag{4}$$

The model allows α and β to vary with x via linear functions (exponential link to ensure positivity):

$$\alpha(x) = \exp(\alpha_0 + \alpha_1 x), \tag{3}$$
$$\beta(x) = \exp(\beta_0 + \beta_1 x). \tag{4}$$

Theorem 1: The relation (4) is a true bivariate probability density function for (X, Y).

Proof

$$\iint f_{X,Y}(x,y)\,dy\,dx = \int_{-\infty}^{\infty} f_X(x)\left(\int_0^1 f_{Y|X}(y|x)\,dy\right)dx$$
$$= \int_{-\infty}^{\infty} f_X(x)\cdot 1\,dx = 1. \tag{5}$$

Since the inner integral equals 1 (property of Beta pdf), the relation satisfies the normalization condition of a valid bivariate probability density function.

2.1.2. Generalization

Suppose we have P bounded responses $Y_1, \ldots, Y_P$. Assume conditional independence among $Y_1, \ldots, Y_P$ given $X = x$:

$$f(y_1, \ldots, y_P|x) = \prod_{j=1}^{P} f_{Y_j|X}(y_j|x).$$

Thus, the full joint density becomes:

$$f(x, y_1, \ldots, y_P) = f_X(x)\prod_{j=1}^{P}\frac{y_j^{\alpha_j(x)-1}(1-y_j)^{\beta_j(x)-1}}{B(\alpha_j(x), \beta_j(x))}. \tag{6}$$

This formulation extends the conditional Beta-Normal model to accommodate multiple responses through a multivariate joint density structure. The 3D plots show how exponential parameterization flexibly controls the shape of the Beta density as a function of the normal covariate (Figure 1).

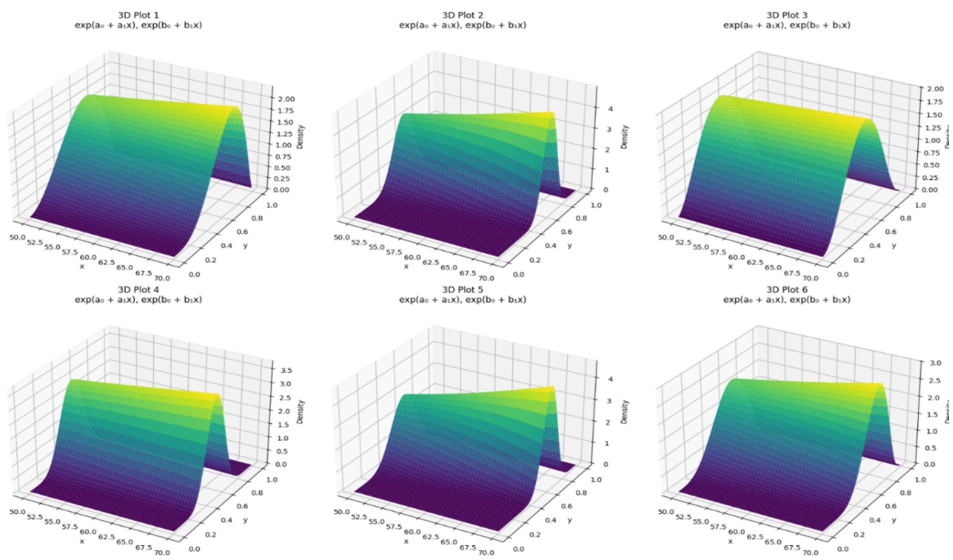

Figure 1: 3D Density Plots of Conditional Beta- Normal Distribution for Various Parameter Values

2.1.3. Properties of CBND

2.1.3.1. Marginal Distribution of Y:. The random variable Y in the joint density Eq. (4) has a marginal distribution given by

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)\, dx.$$

This cannot be integrated analytically in general. Numerical integration was required. The marginal distribution of Y is unimodal and right-skewed, with increasing peak sharpness as parameter values rise. Specifically, let α_0 and α_1 denote the intercept and slope parameters of the first shape function $\alpha(x) = \exp(\alpha_0 + \alpha_1 x)$ of the conditional Beta distribution, while β_0 and β_1 represent the corresponding intercept and slope of the second shape function $\beta(x) = \exp(\beta_0 + \beta_1 x)$.

Table 1 presents the numerical validation of the marginal density of Y, confirming that the integral of the joint density over $x \in (-\infty, \infty)$ equals unity across a wide range of parameter configurations. Table 2, in turn, verifies the normalization of the posterior density $f(x|y)$ under diverse parameter settings. In all 16 tested scenarios (16 mixed-varying), the numerical integrals are effectively 1.0 (up to at least 6 decimal places), demonstrating excellent numerical stability and rigorously confirming that the proposed Conditional Beta-Normal Distribution is a valid probability density function.

The posterior density plot (Figure 4) illustrates the conditional distribution of the loan-to-deposit ratio (LDR) given the liquidity ratio (LR), computed using the estimated model parameters. This reflects the updated belief about LDR after observing LR and further validates the model's coherence.

Table 1: Numerical Integral Varying Parameter Settings

s/n	α_0	α_1	β_0	β_1	Numerical Integral
1	0.5	0.5	0.5	0.5	1.0000
2	1.0	1.0	1.0	1.0	1.0000
3	1.5	1.5	1.5	1.5	1.0000
4	2.5	2.5	2.5	2.5	0.9877
5	3.5	3.5	3.5	3.5	0.9795
6	4.5	4.5	4.5	4.5	0.9770
7	5.5	5.5	5.5	5.5	0.9764
8	7.5	7.5	7.5	7.5	0.9763
9	0.5	1.0	1.5	2.5	0.9999
10	2.5	1.5	1.0	0.5	1.0000
11	1.5	0.5	2.5	1.0	1.0000
12	4.5	2.5	0.5	1.5	1.0000
13	5.5	1.0	3.5	0.5	1.0000
14	1.0	4.5	1.5	3.5	1.0000
15	3.5	5.5	0.5	2.5	0.9999
16	7.5	0.5	1.0	4.5	0.9763

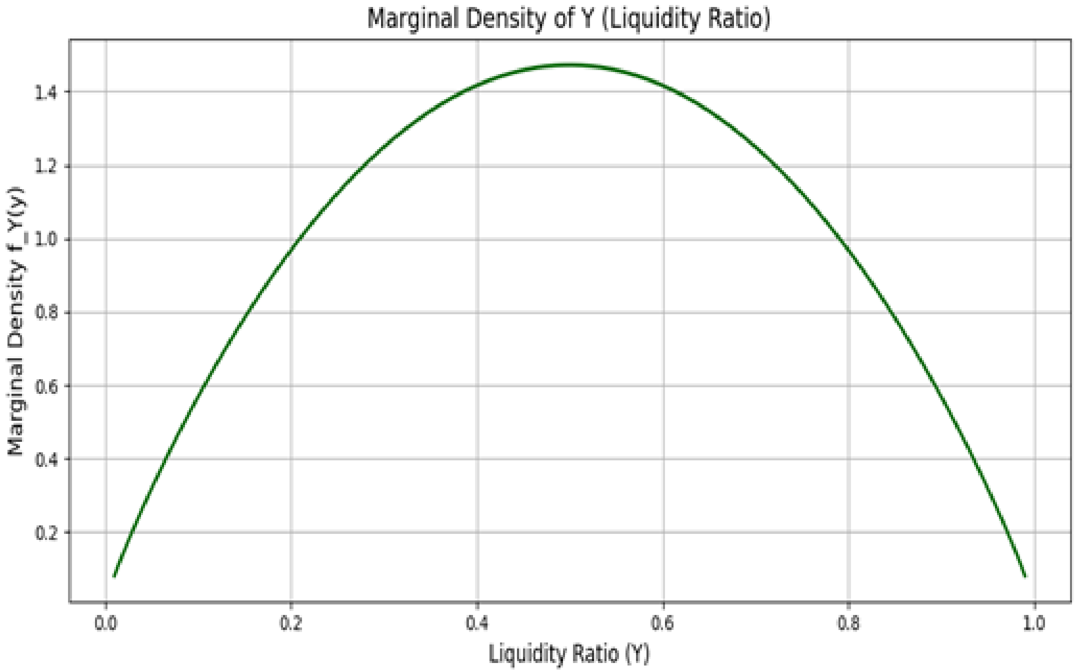

Figure 2: Marginal Distribution of Y

2.1.3.2. Posterior Distribution :. Given $Y = y$, the posterior distribution of X is

$$f(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}. \tag{7}$$

Table 2: Posterior Integral Varying Parameter Settings

S/N	α_0	α_1	β_0	β_1	Posterior Integral
1	0.5	0.5	0.5	0.5	1.00
2	1.0	1.0	1.0	1.0	1.00
3	1.5	1.5	1.5	1.5	1.00
4	2.5	2.5	2.5	2.5	1.00
5	3.5	3.5	3.5	3.5	1.00
6	4.5	4.5	4.5	4.5	1.00
7	5.5	5.5	5.5	5.5	1.00
8	7.5	7.5	7.5	7.5	1.00
9	0.5	1.0	1.5	2.5	1.00
10	2.5	1.5	1.0	0.5	1.00
11	1.5	0.5	2.5	1.0	1.00
12	4.5	2.5	0.5	1.5	1.00
13	5.5	1.0	3.5	0.5	1.00
14	1.0	4.5	1.5	3.5	1.00
15	3.5	5.5	0.5	2.5	1.00
16	7.5	0.5	1.0	4.5	1.00

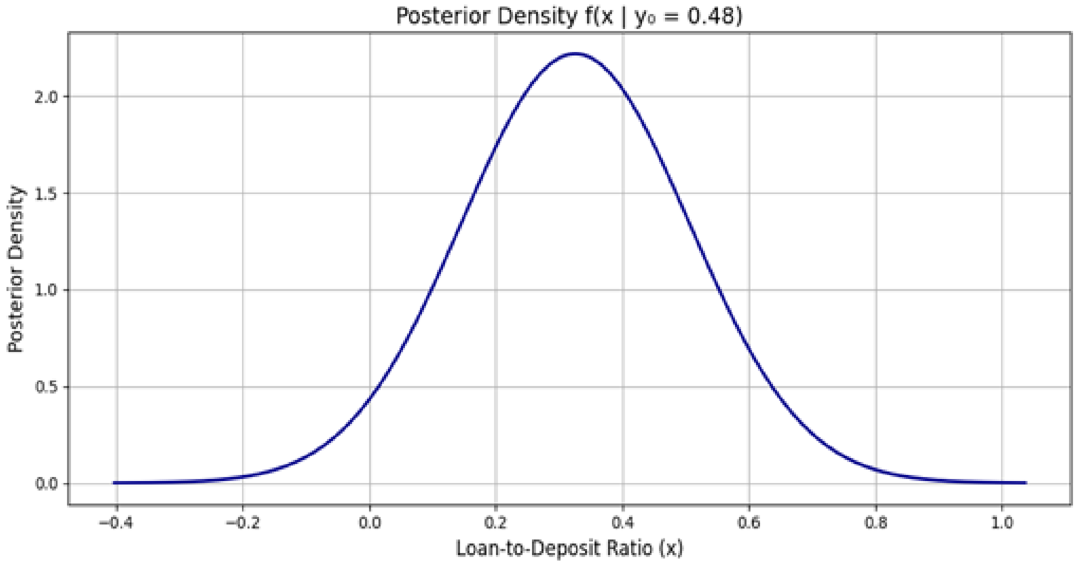

Figure 3: Posterior Density

2.1.3.3. Joint Moment :. The second-order joint moment is

$$E[XY] = \int_{-\infty}^{\infty} \int_{0}^{1} xy f_{X,Y}(x, y)\, dy\, dx = \mu \cdot E[Y]. \tag{8}$$

Higher moments require numerical integration.

Loan-to-deposit ratio is approximately normal; Liquidity Ratio's bounded, right-skewed pattern indicates a Beta distribution (Figure 4). The CBND contour plot fits loan-to-deposit and liquidity ratios well, showing data clusters and their joint distribution clearly.

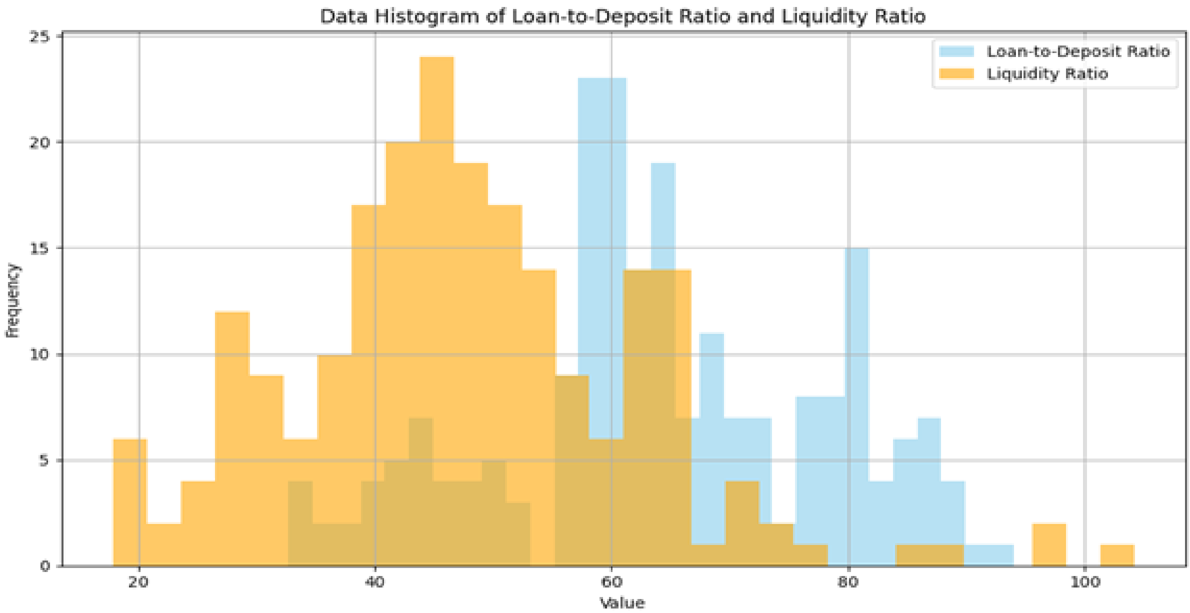

Figure 4: Data Histogram

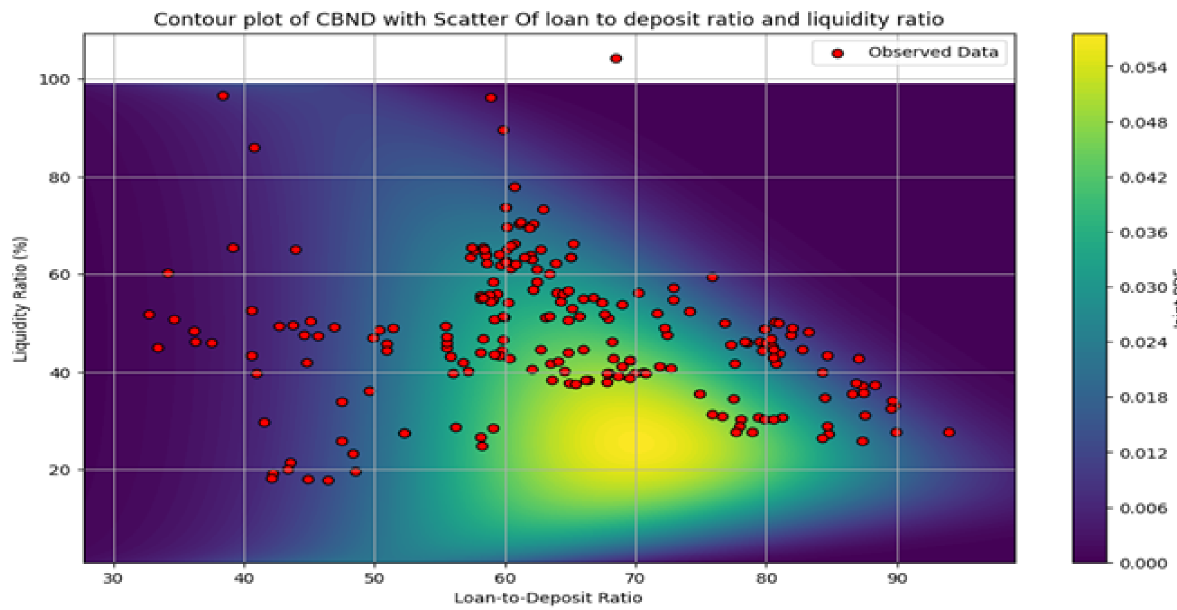

Figure 5: Contour Plot of CBND with Scatter of Loan to Deposit Ratio and Liquidity Ratio

2.1.4. Parameter Estimation

The log-likelihood function is

$$\ell(\theta) = \sum_{i=1}^{n} \Bigg[-\ln(\sigma\sqrt{2\pi}) - \frac{(x_i-\mu)^2}{2\sigma^2} + (\alpha(x_i)-1)\ln y_i + (\beta(x_i)-1)\ln(1-y_i) - \ln B(\alpha(x_i),\beta(x_i)) \Bigg], \tag{10}$$

where $\alpha(x_i) = \exp(\alpha_0 + \alpha_1 x_i)$ and $\beta(x_i) = \exp(\beta_0 + \beta_1 x_i)$.

Closed-form solutions exist for μ and σ:

$$\hat{\mu} = \bar{x}, \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

The remaining parameters are estimated numerically using the L-BFGS-B algorithm with standard errors from the inverse Hessian.

The maximum likelihood estimates are statistically reliable, with all standard errors below 1 (Table 3). The results reveal that liquidity behaviour (captured by the Beta shape parameters) varies systematically with lending intensity: both $\alpha_1 > 0$ and $\beta_1 > 0$ are positive and highly significant. This confirms that the conditional distribution of the liquidity ratio becomes increasingly concentrated as the loan-to-deposit ratio rises, supporting the effectiveness and interpretability of the proposed CBND model.

Figure 6 presents the fitted conditional Beta distributions of LR for different values of LDR using the estimated parameters. As LDR increases, the conditional Beta density of LR visibly narrows and shifts toward higher values, indicating substantially reduced liquidity dispersion among banks when lending activity is intense. This visual evidence corroborates the positive slope estimates for both shape parameters and constitutes the central empirical finding of the study: higher lending levels are associated with tighter and more predictable (less variable) liquidity positions. This pattern has direct and critical implications for liquidity risk management, stress testing, and the design of macro-prudential policies aimed at preserving banking system stability.

Table 3: Maximum Likelihood Estimates (with Standard Error)

Parameter	Estimate	Std. Error
μ	64.2400	0.4120
σ	13.5400	0.2910
α_0	1.8230	0.1560
α_1	0.0125	0.0021
β_0	2.1070	0.1740
β_1	0.0098	0.0023

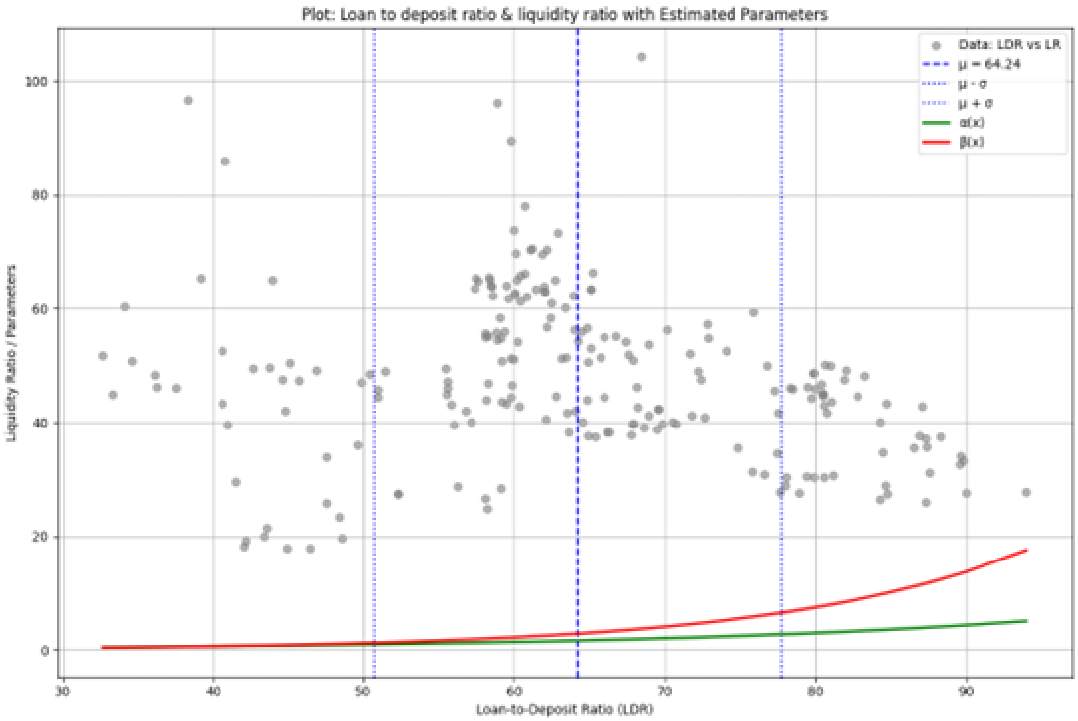

Figure 6: Loan-to-Deposit Ratio and Liquidity Ratio with Estimated Parameters

2.2. *Computation of LDR and LR*

$$\text{LDR} = \frac{\text{Total Loans and Advances}}{\text{Total Deposits}}, \tag{13}$$

$$\text{LR} = \frac{\text{Liquid Assets}}{\text{Total Deposits}}. \tag{14}$$

where LR values exceeded unity, normalization was applied:

$$LR^* = \frac{LR}{1 + MaxLR}. \tag{15}$$

This transformation maintains proportional structure while satisfying the Beta distribution's (0,1) conditions. Liquidity Ratio (LR) can occasionally exceed unity in extreme financial conditions, the Beta distribution was chosen because, for the majority of banking institutions, LR remains bounded within the 0–1 range when expressed as a proportion of deposits covered by liquid assets.

3. Real Data Application

The model was applied to monthly observations of Nigerian banks' LDR and LR from January 2007 to December 2024 (Central Bank of Nigeria, 2024). This period covers major financial cycles including pre- and post-recession years. The LDR exhibited near-normal symmetry (mean = 64.24, SD = 13.54), while the LR variable follows a right-skewed, bounded distribution consistent with the Beta assumption.

4. Results and Discussion

The CBND model's maximum likelihood estimates and corresponding standard errors below one, indicate strong statistical reliability (Table 3). The model achieved a Mean Squared Error (MSE) of 0.6922 (less than 1), confirming a good fit and adequate predictive capability for the financial indicators. Figure 6 illustrates that as LDR increases, LR distributions become more concentrated—suggesting that aggressive lending correlates with tighter liquidity control. This aligns with findings by [1] and [14]. Unlike these approaches that assumed independent marginal relationships, the CBND model captures conditional dependency between LDR and LR. Also, results align with [11] and [7], who applied panel and Beta regression models to Nigerian banking data. Similarly, [4] employed a dynamic copula with time-varying Beta margins to model bounded financial ratios, such as liquidity ratios, and demonstrated that dependence parameters evolve with market volatility. [8] introduced a conditional bivariate model with one bounded and one unbounded component, which is conceptually similar to the approach in the present study, showing that conditional specification better captures asymmetric relationships between LDR and LR variables compared to classical Gaussian copulas. Collectively, these copula and conditional approaches highlight the need for hybrid probabilistic models capable of representing asymmetric, nonlinear dependencies — a gap addressed by the proposed Conditional Beta-Normal Distribution (CBND) structure in this study. Thus, the CBND approach provides an improvement over existing frameworks in both interpretability and modeling precision.

5. Conclusion

A Conditional Beta-Normal Distribution (CBND) structure was developed to jointly model unbounded and bounded financial indicators. Application to Nigerian banking data demonstrated the model's flexibility in capturing nonlinear dependence between the loan-to-deposit and liquidity ratios. Posterior validation confirmed strong fit, with implications for liquidity management and stress testing. Future work may extend this to a Bayesian or multivariate setting for improved inference and uncertainty quantification.

Conflict of Interest

Authors declared there was no conflict of interest

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